Propagation of gravitational waves in a cosmological background

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Why modified gravity

Phenomenological side:

To explain the early and late accelerated expansion of our universe.

Theoretical side:

To understand why GR is unique.

“The best way to understand something is to modify it.”

How to modify gravity

Einstein equation is quite unique.

$$\alpha G_{\mu\nu} + \lambda g_{\mu\nu} = 0$$

[Lovelock, 1971]

Any metric theory of gravity alternative to GR must satisfy (at least):

- extra degrees of freedom (scalar-tensor theories),
- extra dimensions,
- higher derivative terms,
- non-Riemannian geometry,
- non-locality.

Covariant scalar-tensor theories

1915 GR

$$\mathcal{L} = \frac{1}{16\pi G} R$$

1961 Brans-Dicke

[Brans & Dicke, 1961]

$$\mathcal{L} = \phi R - \frac{\omega}{\phi} (\partial\phi)^2$$

1999 *k*-essence

[Chiba, Okabe, Yamaguchi, 1999]

[Armendariz-Picon, Damour, Mukhanov, 1999]

$$\mathcal{L} = g(\phi) R + F(\phi, \partial\phi)$$

2011 Horndeski

[Horndeski, 1974]

[Deffayet, XG, Steer & Zahariade, 2011]

2015 DHOST

[Langlois & Noui, 2015]

[Crisostomi, Koyama & Tasinato, 2016]

In D=4:

$$\begin{aligned}\mathcal{L} = {} & G_2(X,\phi) + G_3(X,\phi)\Box\phi \\ & + G_4(X,\phi)\, R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ & + G_5(X,\phi)\, G^{\mu\nu}\nabla_\mu\nabla_\nu\phi \\ & - \frac{1}{6}\frac{\partial G_5}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right]\end{aligned}$$

with $X = -\frac{1}{2}(\partial\phi)^2$

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2015 DHOST

[Langlois & Noui, 2015]

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Even beyond?

with $X = -\frac{1}{2}(\partial\phi)^2$

Spatially covariant gravity theories

How to introduce a scalar degree of freedom?

Reduction of symmetries → Extra mode(s)

Breaking time diff, respecting only spatial symmetries → 2 tensor + 1 scalar

Two faces of scalar-tensor theories

Gauge fixing: $\phi(t, \vec{x}) \rightarrow t$
(unitary gauge)

Spacetime covariant
Scalar-tensor theories

$$\mathcal{L}(\phi, g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu)$$

Spatially covariant
gravity theories

$$\mathcal{L}(t, N, h_{ij}, R_{ij}, K_{ij}, \nabla_i)$$

Gauge recovering: $t \rightarrow \phi(t, \vec{x})$
(Stueckelberg trick)

[H. Motohashi, T. Suyama, K. Takahashi, 2016]
[A. De Felice, D. Langlois, S. Mukohyama, K. Noui & A. Wang, 2018]

Early examples

2004 Ghost condensation

[Arkani-Hamed, Cheng, Luty & Mukohyama]

2007 Effective field theory of inflation

[Cheung, Creminelli, Fitzpatrick, Kaplan & Senatore]

2009 Hořava gravity

[Horava]

2014 GLPV theory

*[**G**leyzes, **L**anglois, **P**iazza & **V**ernizzi]*

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*[XG, **Phys.Rev. D** 90 (2014) 081501 (**Rapid Communication**)]*

$$S = \int \mathrm{d}t\mathrm{d}^3x\, N\sqrt{h}\mathcal{L}(t, N, h_{ij}, R_{ij}, K_{ij}, \nabla_i)$$

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$$S=\int \mathrm{d}t\mathrm{d}^3x\, N\sqrt{h}\mathcal{L}(t,N,h_{ij},R_{ij},K_{ij},\nabla_i)$$

2 tensor + 1 scalar DoFs with higher derivative EoMs.
[XG, Phys.Rev. D90 (2014) 104033]

Constraint from the gravitational waves

Speed of the GWs:

$$-3 \times 10^{-15} \leq \frac{c_{\mathrm{g}}}{c} - 1 \leq 7 \times 10^{-16}$$

[Phys. Rev. Lett. 119, 161101 (2017), Astrophys. J. 848, L13 (2017)]

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Constraint on the Horndeski/DHOST theory:

$$\begin{aligned}\mathcal{L} = P(\phi, X) &+ Q(\phi, X)\Box\phi \\ &+ G_4(\phi, X)\,{}^4\!R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ &+ G_5(\phi, X)\,{}^4\!G^{\mu\nu}\nabla_\mu\nabla_\nu\phi \\ &- \frac{1}{6}\frac{\partial G_5}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right]\end{aligned}$$

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Constraint on the Horndeski/DHOST theory:

$$\mathcal{L}_{c_T=1} = P(\phi, X) + Q(\phi, X)\Box\phi$$
$$+ f(\phi)^4 R$$

[P. Creminelli, F. Vernizzi, Phys.Rev.Lett. 119 (2017) no.25, 251302]

GWs in a cosmological background

Gravitational waves in a cosmological background:

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Kinetic terms: $K_{ij}K^{ij},$

Potential terms: $R,$

GWs in a cosmological background

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$$S_2 = \int \mathrm{d}t\mathrm{d}^3x\,\frac{a^3}{2}\Big(\mathcal{G}_0\dot{\gamma}_{ij}\dot{\gamma}^{ij} \qquad\qquad \Big)$$

Potential terms: R,

GWs in a cosmological background

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$$S_2 = \int \mathrm{d}t\mathrm{d}^3x\, \frac{a^3}{2}\Big(\mathcal{G}_0\dot{\gamma}_{ij}\dot{\gamma}^{ij} + \mathcal{W}_0\gamma_{ij}\frac{\partial^2}{a^2}\gamma^{ij}\Big)$$

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Potential terms: $R,$ $\boxed{R^2, \quad \nabla_k R_{ij}, \quad \nabla_k\nabla_l R_{ij}, \cdots}$

GWs in a cosmological background

Gravitational waves in a cosmological background:

Kinetic terms: $K_{ij}K^{ij}$,

$$
\begin{aligned}
S_2 \quad = \quad & \int \mathrm{d}t\mathrm{d}^3x \, \frac{a^3}{2}\Big(\mathcal{G}_0 \dot{\gamma}_{ij}\dot{\gamma}^{ij} \\
& +\mathcal{W}_0 \gamma_{ij}\frac{\partial^2}{a^2}\gamma^{ij} + \boxed{\mathcal{W}_1 \epsilon^{ijk}\gamma_{li}\frac{1}{a}\frac{\partial^2}{a^2}\partial_j \gamma^l_k - \mathcal{W}_2 \gamma_{ij}\frac{\partial^4}{a^4}\gamma^{ij} + \cdots}\Big)
\end{aligned}
$$

Potential terms: R, $\boxed{R^2, \quad \nabla_k R_{ij}, \quad \nabla_k \nabla_l R_{ij}, \cdots}$

GWs in a cosmological background

[XG & Xun-Yang Hong, 1906.07131]

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Gravitational waves in a cosmological background:

Kinetic terms: $K_{ij}K^{ij}$, $\boxed{\nabla_k K_{ij}, \quad \nabla_k \nabla_l K_{ij}, \cdots}$

$$S_2 = \int \mathrm{d}t\mathrm{d}^3x\, \frac{a^3}{2}\Big(\mathcal{G}_0 \dot{\gamma}_{ij}\dot{\gamma}^{ij} \boxed{+ \mathcal{G}_1 \epsilon^{ijk}\dot{\gamma}_{li}\frac{1}{a}\partial_j \dot{\gamma}^l_k - \mathcal{G}_2 \dot{\gamma}_{ij}\frac{\partial^2}{a^2}\dot{\gamma}^{ij} + \cdots}$$
$$+\mathcal{W}_0 \gamma_{ij}\frac{\partial^2}{a^2}\gamma^{ij} + \mathcal{W}_1 \epsilon^{ijk}\gamma_{li}\frac{1}{a}\frac{\partial^2}{a^2}\partial_j \gamma^l_k - \mathcal{W}_2 \gamma_{ij}\frac{\partial^4}{a^4}\gamma^{ij} + \cdots\Big)$$

Potential terms: R, R^2, $\nabla_k R_{ij}$, $\nabla_k \nabla_l R_{ij}, \cdots$

Speed of the gravitational waves

Propagation speed of the gravitational waves:

$$\partial_\tau^2 \gamma_{\boldsymbol{k}}^{(s)} + \mathcal{H}\left(2 + \nu^{(s)}\right)\partial_\tau \gamma_{\boldsymbol{k}}^{(s)} + \left(c_{\mathrm{T}}^{(s)}\right)^2 k^2 \gamma_{\boldsymbol{k}}^{(s)} = 0, \quad s = \pm 2$$

Speed of the gravitational waves

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$$\left(c_{\mathrm{T}}^{(s)}(s,k)\right)^2 = \frac{\mathcal{W}_0}{\mathcal{G}_0}$$

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“potential terms”

$$\left(c_{\mathrm{T}}^{(s)}(s,k)\right)^2 = \frac{\mathcal{W}_0 + \boxed{\mathcal{W}_1 \frac{s}{2}\frac{k}{a} + \mathcal{W}_2 \frac{k^2}{a^2} + \mathcal{W}_3 \frac{s}{2}\frac{k^3}{a^3} + \mathcal{W}_4 \frac{k^4}{a^4} + \cdots}}{\mathcal{G}_0}$$

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“kinetic terms”

Speed of the gravitational waves

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$$\left(c_{\mathrm{T}}^{(s)}(s,k)\right)^2 = \cancel{\frac{\mathcal{W}_0 + \mathcal{W}_1 \frac{s}{2}\frac{k}{a} + \mathcal{W}_2 \frac{k^2}{a^2} + \mathcal{W}_3 \frac{s}{2}\frac{k^3}{a^3} + \mathcal{W}_4 \frac{k^4}{a^4} + \cdots}{\mathcal{G}_0 + \mathcal{G}_1 \frac{s}{2}\frac{k}{a} + \mathcal{G}_2 \frac{k^2}{a^2} + \mathcal{G}_3 \frac{s}{2}\frac{k^3}{a^3} + \mathcal{G}_4 \frac{k^4}{a^3} + \cdots}} \quad \boxed{= 1}$$

Effects in the “potential” and “kinetic” terms can get cancelled.

Speed of the gravitational waves

How if $c_T = 1$ in our spatially covariant gravity framework?

Speed of the gravitational waves

How if $c_T = 1$ in our spatially covariant gravity framework?

[XG & Xun-Yang Hong, 1906.07131]

$$
\begin{aligned}
L_{c_{\mathrm{T}}=1} = \; & c_1^{(2,0)}\left(K_{ij}K^{ij}+R\right)+c_2^{(2,0)}K^2 \\
& +c_1^{(3,0)}\left(K_{ij}K^{jk}K_k^i+RK\right)+c_2^{(3,0)}\left(K_{ij}K^{ij}+R\right)K+c_3^{(3,0)}K^3 \\
& +c_1^{(1,2)}\nabla^i\nabla^j K_{ij}+c_2^{(1,2)}\nabla^2 K+c_3^{(1,2)}G^{ij}K_{ij}-\frac{1}{2}\partial_t c_3^{(1,2)}R, \\
& +c_1^{(4,0)}\left(K_{ij}K^{jk}K_k^i+RK\right)K+c_2^{(4,0)}\left(\left(K_{ij}K^{ij}\right)^2+\frac{2}{3}RK^2\right)+c_3^{(4,0)}\left(K_{ij}K^{ij}+R\right)K^2+c_4^{(4,0)}K^4 \\
& +c_1^{(3,1)}\varepsilon_{ijk}\left(\nabla_m K_n^i K^{jm}K^{kn}+\frac{1}{3}\nabla^i K_l^j K^{kl}K\right)+c_2^{(3,1)}\varepsilon_{ijk}\left(\nabla^i K_m^j K_n^k K^{mn}-\frac{2}{3}\nabla^i K_l^j K^{kl}K\right) \\
& +c_1^{(2,2)}\left(\nabla_k K_{ij}\nabla^k K^{ij}-R_{ij}R^{ij}\right)+c_2^{(2,2)}\nabla_i K_{jk}\nabla^k K^{ij}+c_3^{(2,2)}\nabla_i K^{ij}\nabla_k K_j^k+c_4^{(2,2)}\nabla_i K^{ij}\nabla_j K \\
& +c_5^{(2,2)}\nabla_i K\nabla^i K+c_6^{(2,2)}R_{ij}\left(K_k^i K^{jk}-\frac{2}{3}K^{ij}K+\frac{1}{9}h^{ij}K^2\right)+c_7^{(2,2)}R\left(K_{ij}K^{ij}-\frac{1}{3}K^2\right) \\
& +c_1^{(1,3)}\varepsilon_{ijk}R^{il}\nabla^j K_l^k+c_2^{(1,3)}\varepsilon_{ijk}\nabla^i R_l^j K^{kl}+\frac{1}{2}\partial_t\left(c_1^{(1,3)}+c_2^{(1,3)}\right)\varepsilon_{ijk}K_l^i\nabla^j K^{kl} \\
& +c_1^{(0,4)}\nabla^i\nabla^j R_{ij}+c_2^{(0,4)}\nabla^2 R+c_4^{(0,4)}R^2.
\end{aligned}
$$

Speed of the gravitational waves

How if $c_T = 1$ in our spatially covariant gravity framework?

Horndeski with $c_T = 1$:

$$S^{(\mathrm{u.g.})}_{\mathrm{H},c_{\mathrm{T}}=1} = \int \mathrm{d}t\mathrm{d}^3x\, N\sqrt{h}\left[b\left(t\right)\left(K_{ij}K^{ij} - K^2 + R\right) + a_0\, K + d_0\right]$$

Speed of the gravitational waves

How if $c_T = 1$ in our spatially covariant gravity framework?

Parity-violating terms with $c_T = 1$:

$$
\begin{aligned}
\mathcal{O}_1 &= c_1\, \varepsilon_{ijk} \left(\nabla_m K^i_n K^{jm} K^{kn} + \frac{1}{3} \nabla^i K^j_l K^{kl} K \right), \\
\mathcal{O}_2 &= c_2\, \varepsilon_{ijk} \left(\nabla^i K^j_m K^k_n K^{mn} - \frac{2}{3} \nabla^i K^j_l K^{kl} K \right), \\
\mathcal{O}_3 &= \varepsilon_{ijk} \left(c_3 R^{il} \nabla^j K^k_l + \frac{1}{2N} \partial_t c_3 K^i_l \nabla^j K^{kl} \right), \\
\mathcal{O}_4 &= \varepsilon_{ijk} \left(c_4 \nabla^i R^j_l K^{kl} + \frac{1}{2N} \partial_t c_4 K^i_l \nabla^j K^{kl} \right).
\end{aligned}
$$

[XG & Xun-Yang Hong, 1906.07131]

Main message from this talk

- Scalar-tensor theories can be further extended.
- There are more general gravity theories satisfying $c_T = 1$.

Thank you for your attention!